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ABSTRACT

This set of lecture notes is used as a supplemental text for the teaching of fluid dynamics, as one component of a thermodynamics course for engineering technologists. The major text for the course covered basic fluids concepts such as pressure, mass flow, and specific weight. The objective of this document was to present additional fluids information in a relatively short period of time. The notes are divided into four chapters. They are: (1) hydrostatic loads on structures; (2) water flow in pipes; (3) friction losses in pipes and open channels; and (4) orifices. Each chapter includes diagrams, formulae, discussion topics, example problems, and student problems. (TW)

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LECTURE NOTES

APPLIED FLUID MECHANICS

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PREFACE

These lecture notes were written to cover a fluid mechanics course that is combined with a thermodynamics course for engineering technology. It is designed to serve as a text for the students when combined with lectures by the instructor. The material is suitable for covering about one hour of material in a four semester hour course.

The object of the material is to cover the maximum amount of material in a short time. At the same time every effort is made to cover enough theory so that a good basic understand of the material is possible.

It is to be noted that the basic concepts of fluids such as pressure, mass flow, density, specific weight, etc. were covered in the thermodynamics portion. In that study SI and English units were used. In this presentation, only English units are used in the interest of saving time.

The writer wishes to express his thanks to the students who checked the calculations and to Dr. Bill Byers, an associate professor of engineering technology and administrator, who encouraged him in this effort.

Newton D. Gregg, Assistant Professor
Engineering Technology,
University of Central Florida

September, 1986

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jo

1.1 Introduction

It is helpful in understanding hydrostatic loads on structures to review simple beam loadings with uniform, partial uniform, and trapezoidal loadings. Information sought will be reactions and total load. In addition the "center of action" of the loads will be found. This item will have its counterpart in the center of pressure for hydrostatic loads.

1.2 Uniform and trapezoidal loadings on beams

The following units and nomenclature will be used:

w = lbs./ linear foot at a point

R_L = left reaction

R_R = right reaction

T = total load

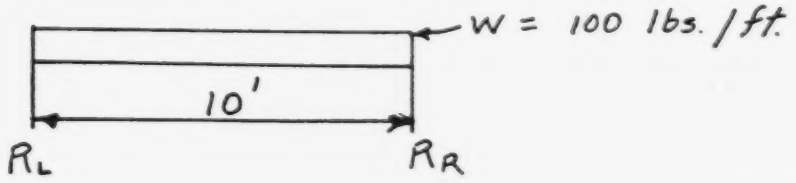
$\bar{x}$ = distance from R_L to the center of action

x_n = distance from R_L to center of action to an individual load

L = load 1

L = load n

Three loading conditions are shown.

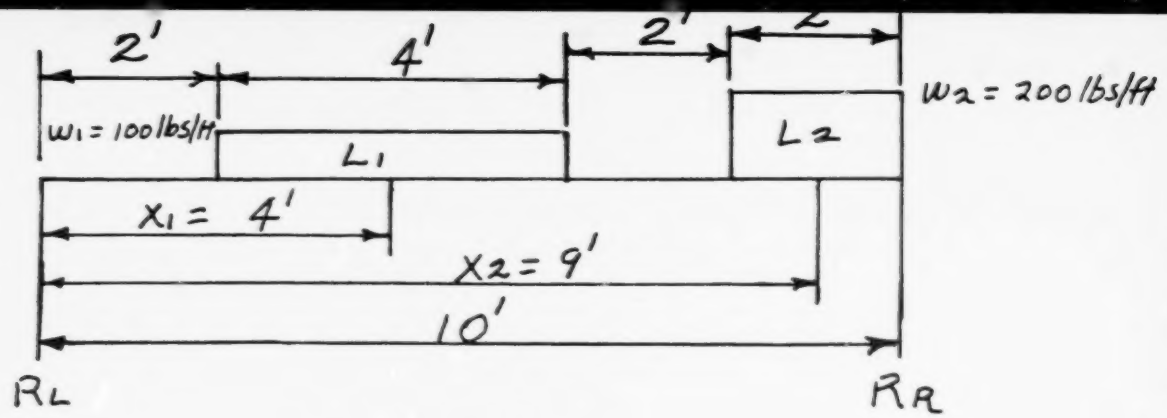


1.2 a

$T = 100 \frac{lbs.}{ft.} \cdot 10\,ft. = 1000\ lbs.$

By symmetry $R_L = R_R = 1000/2 = 500$ lbs.

By symmetry $\bar{x} = 5'$



1.2 b

$L_1 = 4 \times 100 = 400$ lbs $\quad L_2 = 2 \times 200 = 400$ lbs

$T = 400 + 400 = 800$ lbs

$M @ R_R$

$R_L \cdot 10 - 400 \cdot 6 - 400 \cdot 1 = 0$

$10 R_L = 2800$

$R_L = 280$ lbs $\quad R_R = 800 - 280 = 520$ lbs.

Check: $M @ R_L \quad -400 \cdot 4 - 400 \cdot 9 + R_r \cdot 10 = 0$

$10 R_R = 5200 \quad R_R = 520$ lbs

The sum of the individual loads multiplied by their centers of action will equal the total load multiplied by its center of action.

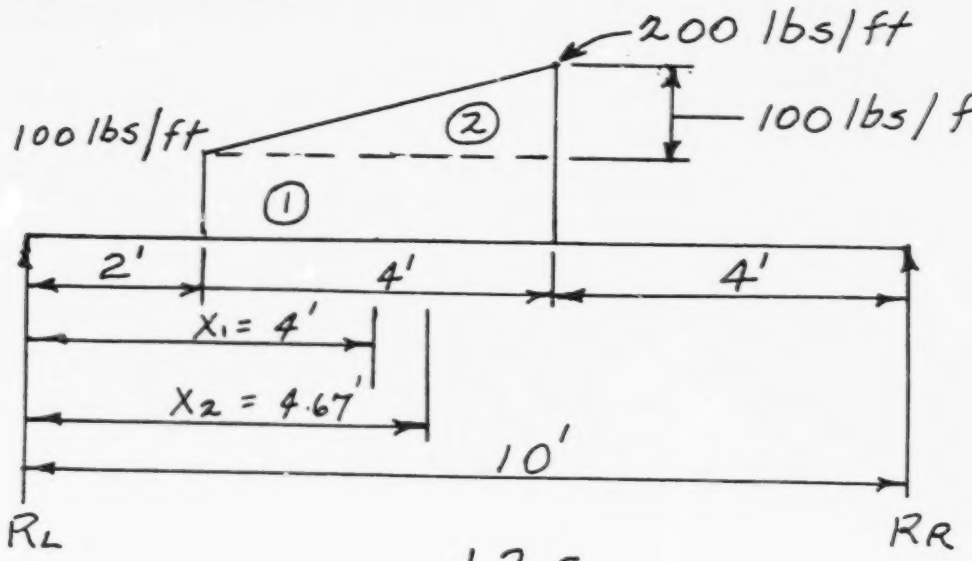
$\bar{X} \cdot 800 = 400 \cdot 4 + 400 \cdot 9$

$800 \bar{X} = 5200$

$\bar{X} = 6.5'$

2

Trapezoidal load is divided into two loads. Centers of action of the loads are shown. C_g of triangle is 1/3 h.



1.2 C

$L_1 = 4 \times 100 = 400$ lbs. $L_2 = (4 \times 100) .5 = 200$ lbs.

$T = 400 + 200 = 600$ lbs

M@ R_R

$R_L \cdot 10 - 400 \cdot 6 - 200 \cdot 5.33 = 0$

$R_L = 3466 \div 10 = 346.6$ lbs.

$R_R = 600 - 346.6 = 253.4$ lbs.

$\bar{X} \cdot 600 = 400 \cdot 4 + 200 \cdot 4.67$

$\bar{X} = (1600 + 934) \div 600 = 4.22'$ from R_L

1.3 Hydrostatic forces on flat surfaces

Symbols:

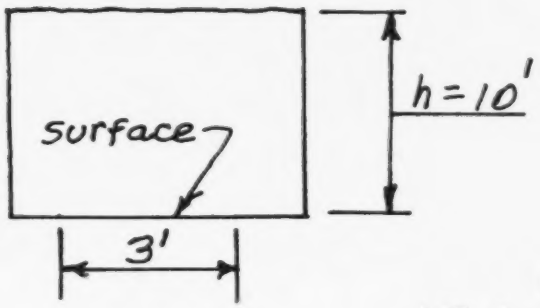
P = pressure in lbs. / ft^2

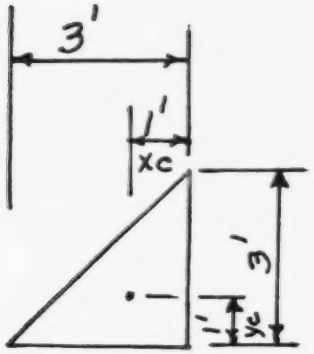
T = total load in lbs.

x_c and y_c = centers of pressure

h = depth in ft.

A = area in ft.





1.3 a PLAN VIEW OF SURFACE

At h depth pressure is uniform at 62.4 h lbs/ ft. Load on any surface will be P · A = lbs. Center of pressure will be at centroid of area.

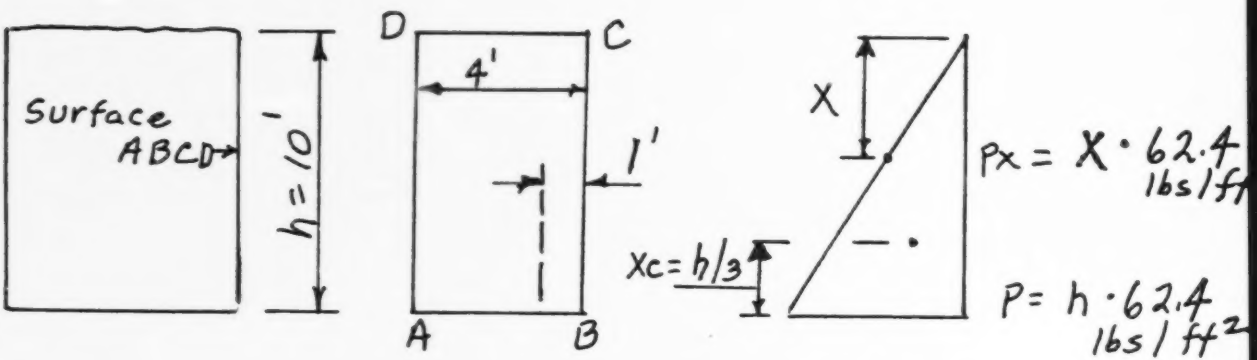
$P = 10 \cdot 62.4 = 624$ lbs./ft^2

$A = 3 \cdot 3(.5) = 4.5$ ft^2

$T = 4.5 \cdot 624 = 2808$ lbs.

1.4 Hydrostatic forces on sides of vertical surfaces

Loads on rectangular areas will be determined for one foot of width. Center of pressure will also be determined by using a one foot width. Total load can be determined by multiplying by the width. Pressure load varies as the depth h at any point. T_1 is load for one foot of width. If a surface has zero pressure at one end the loading is triangular. If both ends have pressure the loading is trapezoidal. Examples of both types of loading are shown.



LOADING

1.4 a

P at AB = 10 · 62.4 = 624 lbs/ft²

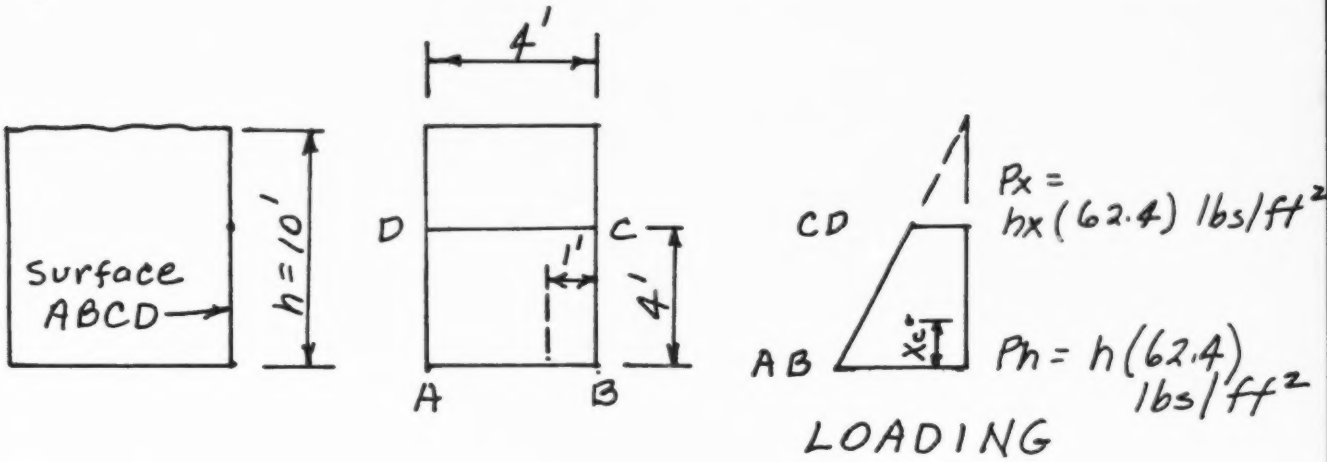
P at DC = 0.

Xc = 10/3 = 3.33' from AB

T_1 = 62.4 · 10(0.5) = 3120 lbs.

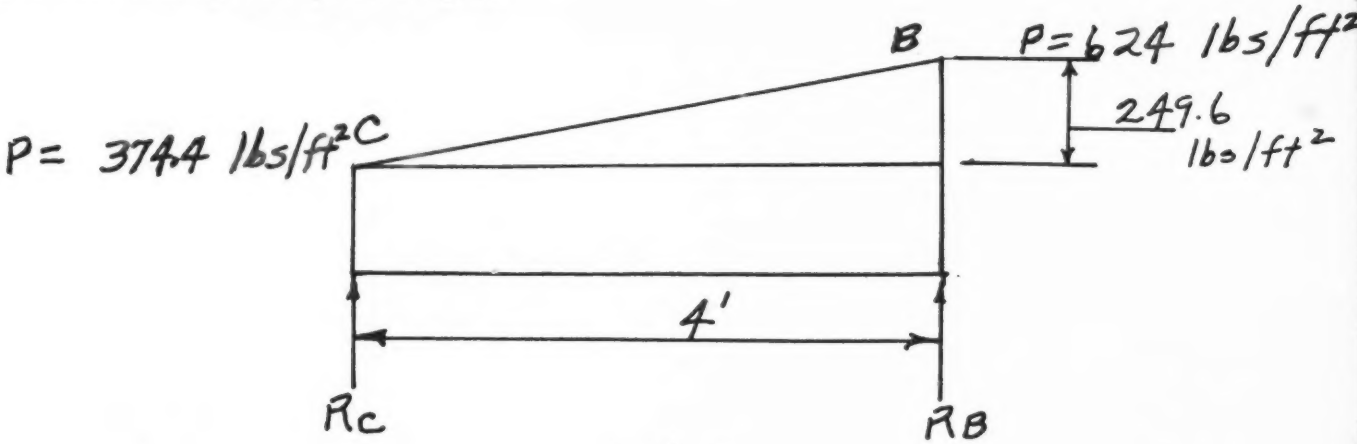
T = 3120 · 4 = 12,480 lbs

Determine loading for area ABCD. Find center of pressure. Determine reactions at top and bottom for a one foot wide section, assuming it is supported at these points. Actual support of an area under load will depend on support conditions but this calculation will be helpful in understanding material in the next section.



1.4 b

T, load can be found by dividing loading into two loads. This will be convenient for finding x_c. If only load were desired it could be found by using the average pressure multiplied by the length.



1.4 C

6

$P@C = 6 \cdot 62.4 = 374.4 \text{ lbs/ft}^2$

$= 374.4 \text{ lbs/ft (1' width)}$

$P@B = 10 \cdot 62.4 = 624 \text{ lbs/ft}$

$L_1 = 374.4 \quad 4 = 1497.6 \text{ lbs}$

$L_2 = 249.6 \cdot 4(.5) = 499.2 \text{ lbs}$

$T_1 = 1497.6 + 499.2 = 1996.8 \text{ lbs}$

m@C

$1996.8 \cdot X_C = 1497.2 \cdot 2 + 499.2 \cdot 4(.5)$

$X_C = 4333.1 / 1996.8 = 2.17'$

from C

$T = 1996.8 \cdot 4 = 7987.2 \text{ lbs}$

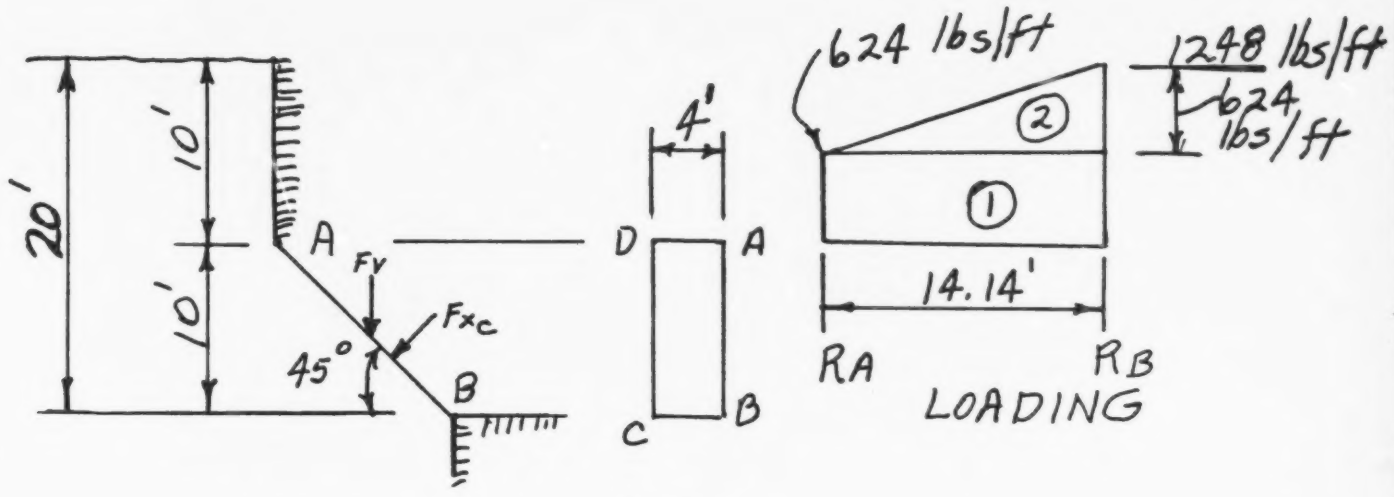
m@ Rc.

$1996.8 : 2.17 - R_B \cdot 4 = 0$

$R_B = 1083 \text{ lbs} \qquad R_C = 1996.8 - 1083 = 913.8 \text{ lbs}$

1.5 Hydrostatic loads on inclined surfaces

Fluid pressure loads are always perpindicular to the surface under load. This the basic nature of fluid loads as stated in Pascal's law. Pressure at ends can be found by determination of heads at these points. Methods used in previous sections can be used to find loads and center of pressure. To show how these loads and gravity loads act on the structure studies will be made to determine balanced moments. One end will be assumed to be hinged and loads will be sought that will put the structure in equilibrium, that is, have the moments balanced about the hinged end. This assumes that there are no other reaction producing forces contributing to these moments.



1.5a

Gate ABCD is four ft. wide and weighs 8,000 lbs. It is under a hydrostatic load as shown. Find the center of pressure and total load on the gate. Assuming gate is hinged at B determine load at x_c to balance moment, neglecting gravity load. When gravity load is included determine what additional vertical force will be required to balance moment?

$P_A = 10 \cdot 62.4 = 624 \text{ lbs/ft}^2$

$P_B = 20 \cdot 62.4 = 1248 \text{ lbs/ft}^2$

$L1 = 624 \cdot 14.14 = 8,823 \text{ lbs}$

$L2 = 624 \cdot 14.14(0.5) = 4411 \text{ lbs}$

$T_1 = 8,823 + 4,411 = 13,234 \text{ lbs}$

$T = 13,234 \cdot 4 = 52,936 \text{ lbs}$

Xc m @ A

$$Xc \cdot 13,234 = \underset{(1)}{8,823 \cdot 7.07} + \underset{(2)}{4,411(14.14 \cdot 0.67)}$$

$Xc = 7.87$ from A

F_{xc} is 52,936 lbs. This counter clockwise moment balances the clockwise moment.

F_v and gravity load of gate have an arm of 5 ft. about B.

$$Fv \cdot 5 + 8,000 \cdot 5 = 52,936(14.14 - 7.87)$$

$$Fv \cdot 5 = -40,000 + 331,908$$

$$Fv = 291,908/5 = 58,382 \text{ lbs.}$$

1.6 Hydrostatic loads on non rectangular areas

Symbols:

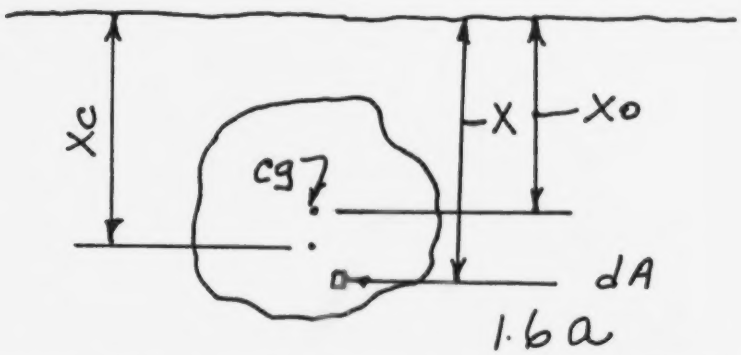
x_o = distance from water surface to c_g

x_c = distance to center of pressure

P = pressure load in lbs.

A = area

x = distance from surface to elemental area



1.6a

Vertical surface above has its c_g located a distance x_o from the surface. Derive expressions for load and center of pressure.

$$dP = dA \cdot X \cdot w \qquad P = w \int dA \cdot X$$

$dA \cdot X$ = statical moment $\qquad P = w A \cdot X_o$

The expression for P shows that the total pressure on an immersed body is the product of its area , the specific weight, and the head on the center of gravity.

Use the same incremental quantity and multiply it by a constant x_c. This term is equated to an incremental value of pressure multiplied by its distance from the surface. This distance is the elemental "x_c". When the entire term is integrated it will equal the integral of x multiplied by the integral of the incremental pressure.

$$X_c \int dP = \int dA \cdot w \cdot X \cdot X$$

$$X_c \int dA \cdot \cancel{w} \cdot X = \int dA \cdot \cancel{w} \cdot X^2$$

$\int dA \cdot X = A \cdot X_o$ statical moment

$\int X^2 dA$ = Moment of Inertia = I_x

$X_c (A \cdot X_o) = I_x$

$X_c = I_x / A \cdot X_o$

I_o = moment of inertia about centroid

$I_x = I_o + X_o^2 A$ (transfer formula)

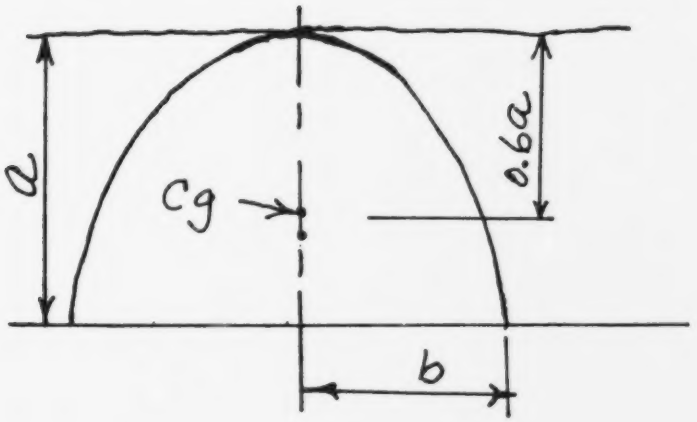
$$X_c = \frac{I_o + X_o^2 \cdot A}{A \cdot X_o}$$

$$X_c = \frac{I_o}{A X_o} + X_o$$

$X_c - X_o = I_o / \cancel{A \cdot X_o}$

This quantity is the distance from the centroid to the center of pressure. The center of pressure must be below the centroid since all quantities on right must be positive.

11



$I_o = 16/175\ a^3 b$

$X_o = 0.6a$

$A = 1.333\ ab$

1.6 b

Shown above is a parabola placed below the surface as indicated. Derive an expression for the value of x_c .

$$X_c - X_o = I_o / A \cdot X_o$$

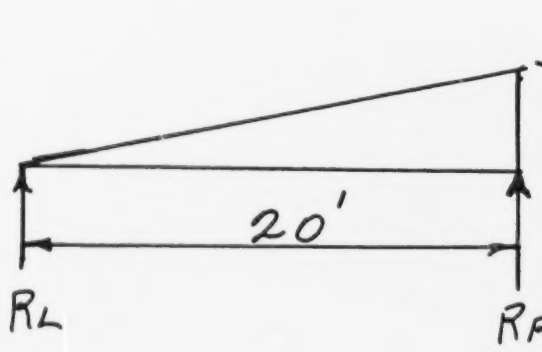
$$X_c - 0.6a = \frac{0.0914\ a^3 b}{1.333\ ab(0.6a)}$$

$$X_c = 0.6a + 0.114a = 0.714a$$

12

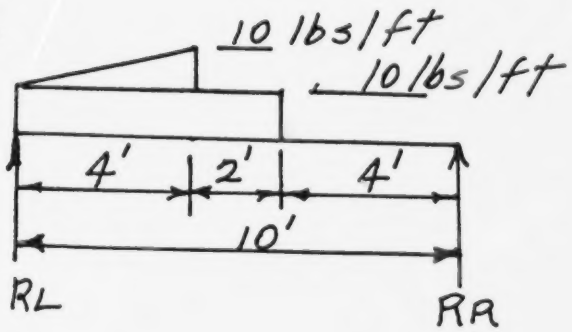
For problems 1.1 through 1.4 find (1) individual loads, (2) total load, (3) reactions, and (4) where total load is acting.

1.1



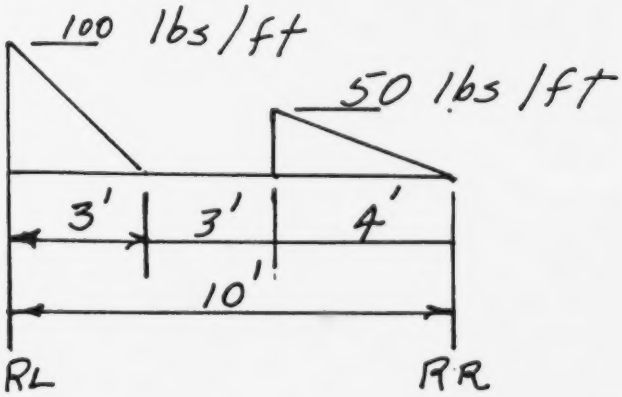
(4) = 6.67' from Rr

1.2



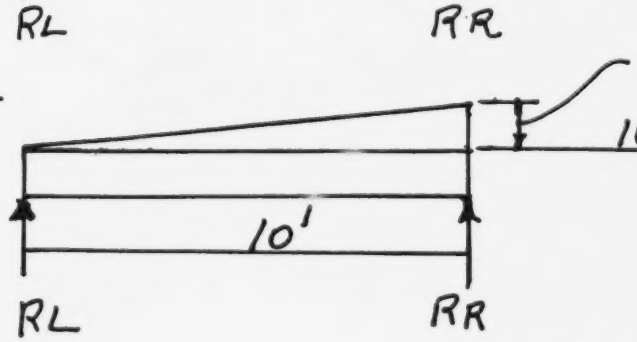
Xc = 2.92' from RL

1.3



Xc = 3.53' from RL

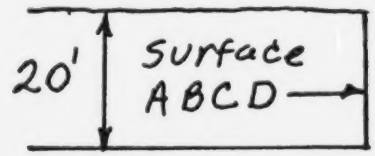
1.4

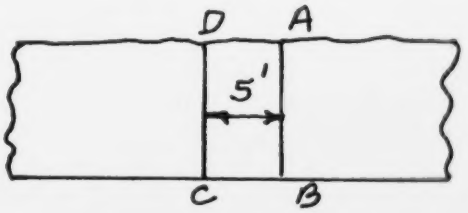


Xc = 5.56' from RL

1.5 (a) What is the total load on a manhole in the bottom of a tank if the pressure gage reads 4.333 psi ? The manhole is 2 ft in diameter. (b) What is the head in feet on the manhole? (b) $h = 10'$

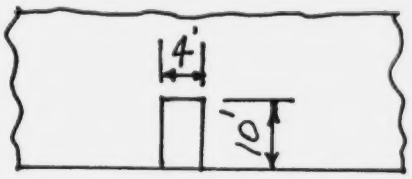
1.6





A 5' wide gate is under a 20' head as shown. Find (a) the total load and (b) where load is acting. $T = 62{,}000$ lbs

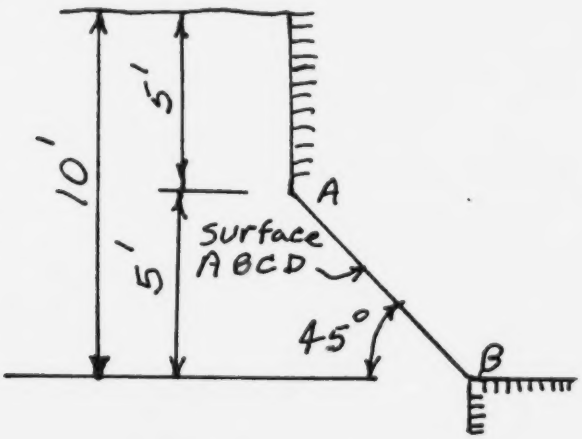
1.7

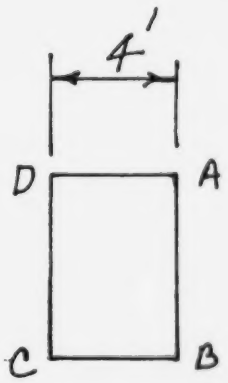


$x_c = 4.44'$ from bottom

Find (a) load on 4 x 10 door and (b) where x_c is located.

1.8

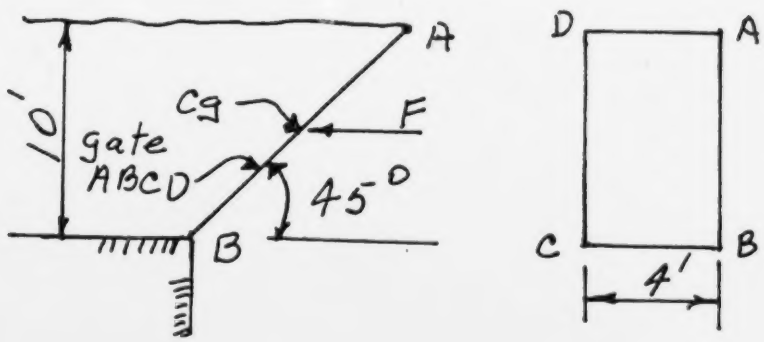




The gate ABCD is hinged at BC and is 4 wide. What vertical force, applied at the center of gravity of the 4,500 lbs gate will keep it in equilibrium?

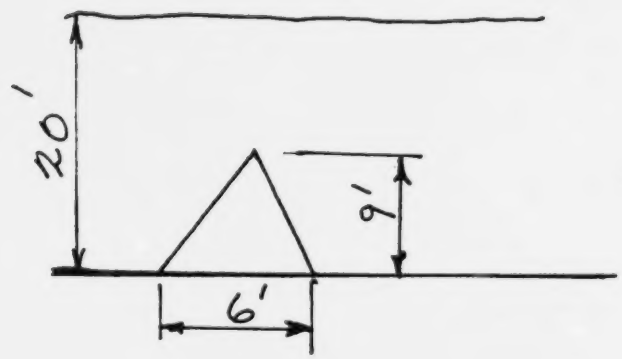
ans. 12,120 lbs

1.9



A gate weighing 8,000 lbs is hinged at BC. What horizontal load will keep the gate in equilibrium? $X_c = 4.71'$ from BC.

1.10



I_o for triangle is $\frac{bd^3}{36}$

The triangle is in a vertical position. Find (a) total pressure on either side, and (b) the center of pressure. $X_c - X_o = 0.263'$

CHAPTER II

Water Flow in Pipes

2.1 Basic Mass Flow Review

Symbols

w = weight of water-- lbs/ft^3

$\dot{W}$ = flow of water in lbs/sec

A = area of pipe in ft^2

v = velocity of flow in ft/sec

Q = volume flow in ft/sec

$$Q = A \cdot V = \frac{ft^2 \cdot ft}{sec} = \frac{ft^3}{sec}$$

$$\dot{W} = w \cdot Q = \frac{lbs}{ft^3} \cdot \frac{ft^3}{sec} = \frac{lbs}{sec}$$

If quantities are expressed in other units such as inches, minutes and hours it is usually desirable to convert to the units shown above. The symbols used are ones used in water systems in the United States. Other fluid systems often use somewhat different symbols as do scientific texts. Some fluid mechanics engineering texts use the Internationsl Systems of Units (SI) as well as the particular units shown above. These units are, of course, taken from the English system. However, some simplication is possible for water systems since constant temperature (and therefore viscosity and specific weight) is assumed and weight is a force quantity. The basic unit for energies of water (head- in feet) lends itself well to this system. However, its relation to basic units for energy and work must be kept in mind and this is emphasized in this presentation.

2.2 Three Energies of a One Pound Load

Symbols

h_e = elevation head in ft. = h·w = h

h_p = pressure head in ft. = p/w = p/62.4

h_p = pressure head in ft. = p/w·144 p(2.31) when p is in psi

h_v = velocity head in ft. = $v^2/2g$

g = acceleration due to gravity = 32.2 $ft./s^2$

v = velocity in ft./s

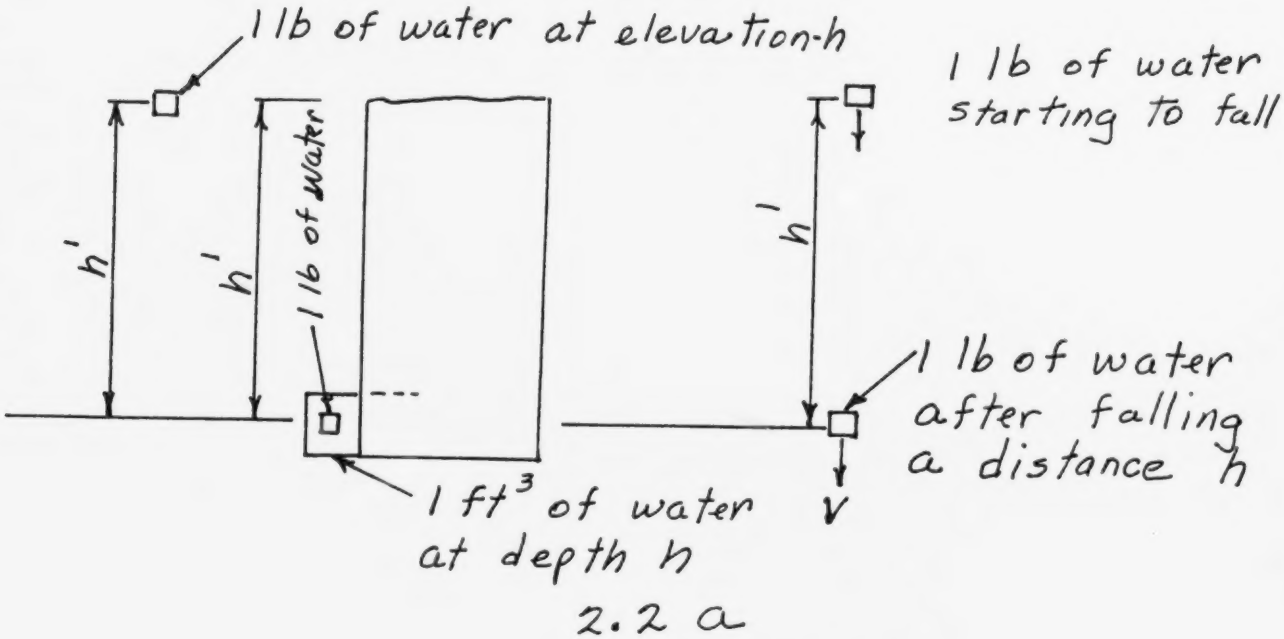
h = elevation distance or total head in ft.

z = elevation distance in ft.

The first chapter dealt with hydrostatic loads and how these loads were applied to surfaces under varying conditions. This chapter deals with energy. They are first considered in a general fashion and then specifically for flow in pipes.

The energies considered are all called head. They are elevation head, velocity head, and pressure head. They all have units of feet. One may say a velocity head of $10'$, a pressure head of $10'$, or an elevation head of $10'$. This can be confusing. It must be kept in mind that what is meant is ft. lbs/lb . Thus the lbs do cancel out but the ft. lbs/lb is technically the correct unit to show that it is an energy. As the units imply one lb. is being used. If total energy is desired the term must be multiplied by the total number of lbs.

The three energies are expressed as h·w, $1/2\ mv^2$, and pv for potential, kinetic, and pressure energies respectively. Expression for these basic terms in terms of the "one pound load" are developed.


1 lb of water at elevation h
1 lb of water starting to fall
h'
h'
1 lb of water
h'
1 lb of water after falling a distance h
1 ft³ of water at depth h
V

2.2 a

Elevation head

In the case of elevation head the expression is h w. w is the unit for pound of water and in this case has a value of 1. Therefore the term h is the value of the head, its units being ft lbs/lb. At 10' the head is 10'.

Pressure head

Pressure at a depth of h is h w in lbs/ft . One ft volume of water is considered. When this pressure is multiplied by the volume:

$$h \cdot w \cdot v = p \cdot v \text{ or } p \cdot 1 = \frac{\text{lbs } \cancel{\text{ft}^3}}{\text{ft}^2} = \text{ft} \cdot \text{lbs}$$

This can be seen as the work required to "push" a ft of water into a tank of water at depth h. But the quantity shown is for 1 ft of water. To obtain value for 1 lb divide by 62.4 since there are 62.4 lbs of water in a cubic ft of water. For one lb of water: h = p/62.4. Example for a pressure head at 10' is:

$$P = h \cdot w \cdot 1 = (10 \cdot 62.4) = 624 \cdot \text{ft lbs}$$

$$\frac{P}{624} = \frac{10 \cdot 62.4}{62.4} = 10 \text{ ft} \cdot \frac{\text{lbs}}{\text{lb}} = 10'$$

P/w is the general expression for pressure head. If w were the specific weight of any fluid the expression would be valid for other fluids as well. For most pipe flow considerations pressure is stated in psi. When psi is used the p/62.4 expression must be multiplied by 144. The pressure head expression is then: $\frac{P}{62.4} \times 144 = 2.31P$.

Velocity head

If the one lb of water is allowed to fall it will have a velocity of $\sqrt{2gh}$ after it has fallen a distance of h feet. This is taken from the basic equations for motion. The potential energy has become kinetic energy.

$$Ke = \frac{1}{2} m v^2$$

$$Ke = \frac{1}{2} \frac{w}{g} v^2$$

For the one lb load:

$$Ke = \frac{1}{2} \frac{(1)}{g} \left(\sqrt{2gh}\right)^2 = \frac{1}{2} \frac{1}{g} 2gh = h$$

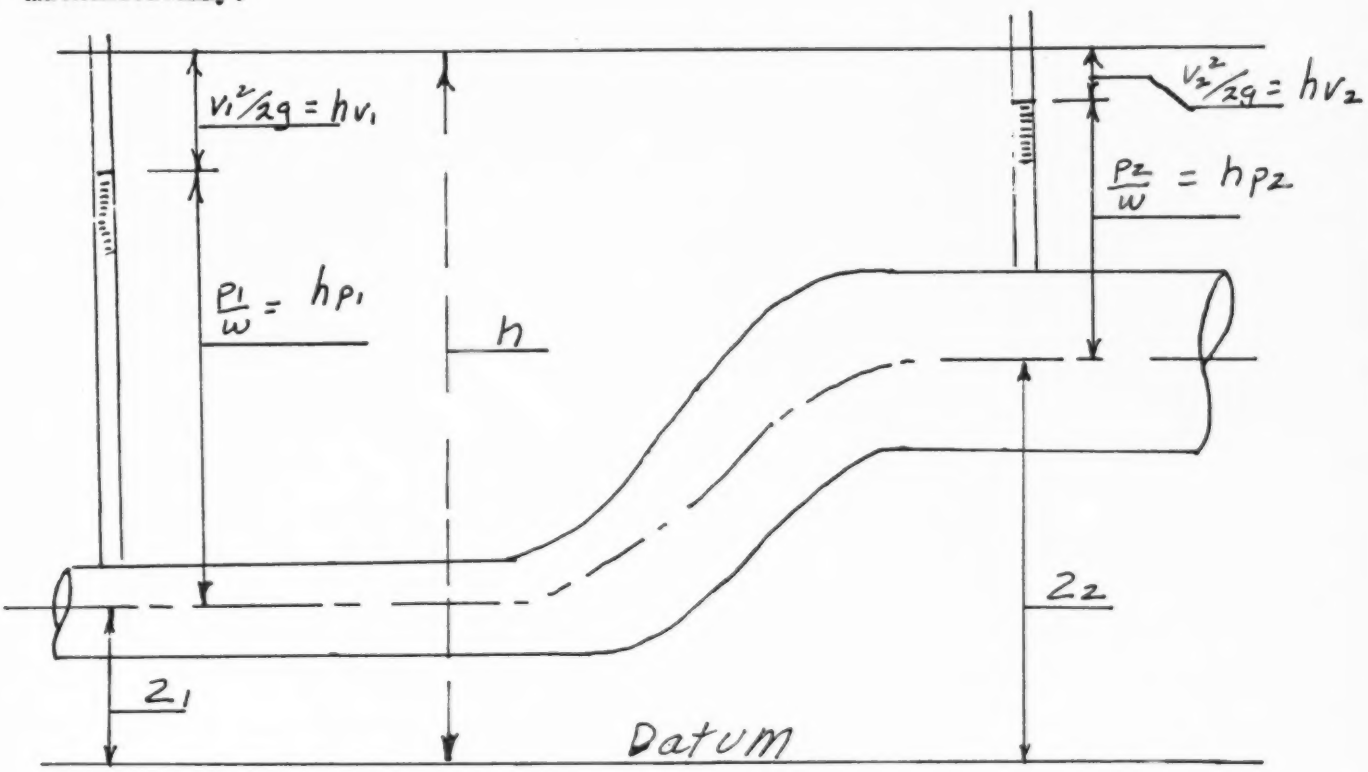
$$hv = h$$

$$hv = \frac{v^2}{2g} \quad \text{in ft or} \quad \frac{\text{ft lbs}}{\text{lb}}$$

The kinetic energy is h and if the h is 10 ft. the velocity head is 10 or 10 ft $\frac{\text{lbs}}{\text{lb}}$. Actually the problems will not be dealing with a"fallen" mass but a lb of water having a particular velocity. The expression $v^2/2g$ is used to determine the velocity head.

2.3 Bernoulli's Equation

Bernoulli's equation states that for liquid flow in a pipe having three values of energy with a total h, h being the sum of energies or "heads" as previously expressed, will remain constant when these energies change because of changing conditions, providing there are no friction losses. Bernoulli's equation is shown graphically and mathematically.



2.3a

$$Z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{w} + \frac{V_2^2}{2g}$$

In the problems Q will be the term for flow in ft^3/sec. $Q = A \cdot V$

$\dot{W}$ will be flow in lbs/sec. $\dot{W} = Q \cdot w$

EXAMPLE PROBLEMS

E 2.3.1 Find head for the following conditions:

(a) at $h = 60'$ $\quad h_h = 60 \cdot w = 60 \cdot (1) = 60 \text{ ft} \cdot \frac{\text{lbs}}{\text{lb}} = 60'$

(b) $V = 10 \text{ fps}$ $\quad h_v = V^2/2g = 10^2/64.4 = 1.55 \text{ ft} \cdot \frac{\text{lbs}}{\text{lb}} = 1.55'$

(c) $P = 45 \text{ psi}$ $\quad h_p = 2.31(45) = 103.9 \text{ ft} \cdot \frac{\text{lb}}{\text{lb}} = 103.9'$

E 2.3.2 Given $D = 3''$ $V = 10 \text{ fps}$ $P = 3 \text{ psi}$

Find A, Q, V_h, total energy passing a point

(a) $A = \pi \left(\frac{.25}{4}\right)^2 = 0.049 \text{ ft}^2$

(b) $Q = AV = 0.049 \times 10 = 0.49 \frac{\text{ft}^3}{\text{sec}}$

(c) $V_h = V^2/2g = \frac{10^2}{64.4} = 1.55 \text{ ft} \cdot \frac{\text{lbs}}{\text{lb}} = 1.55'$

$h = P \cdot 2.31 + V^2/2g = 3(2.31) + 1.55 = 8.48'$

$\dot{w} = Q \cdot w = 0.49 \cdot 62.4 = 30.58 \text{ lb/sec}$

(d) Total energy $= 8.48 \text{ ft} \cdot \frac{\text{lb}}{\text{lb}} \cdot 30.58 \frac{\text{lb}}{\text{sec}}$

$= 259.32 \text{ ft} \cdot \frac{\text{lb}}{\text{sec}}$

$= 0.47 \text{ hp}$

E.2.3.3

Given $z_1 = 0$ $P_1 = 30$ psi V_1 12 fps

$V_2 = 6$ fps $z_2 = 10'$ $D_1 = 6''$

Determine D_2 and P_2.

(a) $A_1 V_1 = A_2 V_2$

$A_1 = \pi \frac{(.5)^2}{4} = 0.196 \text{ ft}^2$

$(0.196)\,12 = A_2 \cdot 6$

$A_2 = 0.392 \text{ ft}^2$

$\frac{\pi}{4} \cdot (D_2)^2 = 0.392$

$D_2 = 0.706 \text{ ft} = 8.48''$

(b) $z_1 + P_1(2.31) + \frac{V_1^2}{2g} = z_2 + P_2(2.31) + \frac{V_2^2}{2g}$

$0 + 30(2.31) + \frac{(12)^2}{64.4} = 10 + P_2(2.31) + \frac{6^2}{64.4}$

$2.31\,P_2 = 60.98$

$P_2 = 26.4$ psi

2.4 Friction and Work Done by Water in Relation to Bernoulli's Equation

Bernoulli's equation is correct if there are no friction losses and if no work is done by the water. For short distances of pipe the equation gives good results. The amount of friction losses will also depend on the nature of the pipe, that is, the type of material and the age of the pipe. There are methods to estimate these losses and they will be explored in the next chapter. In any case the imbalance in the equation will be the friction losses, work done, or both. This chapter assumes that only one of these quantities is not known. If all of the other quantities are known then this value can be determined. Mathematically the Bernoulli equation is modified as follows:

$$Z_1 + P_1/w + V_1^2/2g = Z_2 + P_2/w + V^2/2g + LH$$

LH is lost head. It also could be work done when this term is involved. Examples of both types of "deltas" are shown.

EX 2.4.1

2.4a

Given:

$D_1 = 6''$ $D_2 = 1'$ $Q = 3\ ft^3/sec$ $P_1 = 20\ psi$

$P_2 = 5\ psi$ Determine lost head.

$A_1 = \pi(0.5)^2/4 = 0.196\ ft^2$

$A_1V_1 = Q = 3\ ft^3/sec = A_2V_2$

$0.196\ V_1 = 3$ $V_1 = 15.3\ ft/sec$

$A_2 = \pi \cdot (1)^2/4 = 0.785\ ft^3$

$0.785(V_2) = 3$

$V_2 = 3.82\ ft/sec$

$Z_1 + P_2(2.31) + V_1^2/2g = Z_2 + P_2(2.31) + V_2^2/2g + LH$

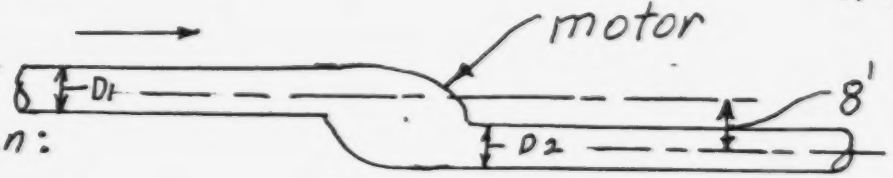
$0 + 20(2.31) + \frac{(15.3)^2}{64.4} = 10 + 5(2.31) + \frac{(3.82)^2}{64.4} + LH$

$46.2 + 3.63 = 10 + 11.55 + 0.23 + LH$

$LH = 28.05'$

EX. 2.4.2

2.4b



Given:

$D_1 = 7''$ $D_2 = 1'$ $P_1 = 100\ psi$ $P_2 = 20\ psi$

$Q = 6\ ft^3/sec$ Determine total work done by the motor.

$A_1 = \pi(0.58)^2/4 = 0.264\ ft^2$

$A_2 = \pi(1)^2/4 = 0.785\ ft^2$

$A_1V_1 = Q = 6\ ft^3/sec$

$0.264\,(V_1) = 6$

$V_1 = 22.73$ ft/sec

$A_2V_2 = 6$ ft^3/sec

$0.785\,V_2 = 6$ ft^3/sec

$V_2 = 7.64$ ft/sec

$Z_1 + P_1(2.31) + V_1^2/2g = Z_2 + P_2(2.31) + V_2^2/2g + WK$

$8 + 100(2.31) + (22.73)^2/64.4 = 0 + 20(2.31) + \frac{(7.64)^2}{64.4} + WK$

$8 + 231 + 8.02 = 46.2 + 0.91 + WK$

$WK = 200.01' = 200.01$ ft·lbs/lb

$Q = 6$ ft^3/sec

$\dot{w} = Q \cdot w = 6(62.4) = \frac{ft^3}{sec} \cdot \frac{lbs}{ft^3} = 374.4$ lbs/sec

Total energy $= 374.4(200.01) = ft\,\frac{lbs}{lb} \cdot \frac{lb}{sec}$

$= 74,880$ ft lbs/sec

$= 136.15$ HP

Homework

2.1 Find head for the following conditions: (express in ft·lbs/lb)

(a) at 100' elevation

(b) $V = 10$ fps

(c) $P = 100$ psi

(d) $P = 624$ psf

(e) $V = 500$ fpm

(f) Elev. = 100 meters (convert to English)

2.2 Given: pipe diameter = 5" $V = 5$ fps

Determine:

(a) Q

(b) Velocity head

(c) Total energy passing a point

Pipe Flow Problems

2.3 Given: $P_1 = 50$ psi $\quad V_2 = 5$ fps

$Z_1 = 10'$ $\quad Z_2 = 20'$

$V_1 = 10$ fps $\quad D_1 = 6"$

Determine P_2, Q, and $\dot{w}$

Answers: $\dot{w} = 122.3$ lbs/sec

$Q = 196$ ft^3/sec

$P_2 = 46.17$ psi

2.4 Given: $Z_1 = 130'$ $\quad V_1 = V_2 = 10$ fps

$P_1 = 50$ psi $\quad P_2 = 100$ psi

$Q = 7.85$ ft^3/sec

Determine: Z_2, D_1, D_2 and $\dot{w}$

Answers: $Z_2 = 14.5'$

$D_1 = D_2 = 1'$

$\dot{w} = 489.8$ lbs/sec

2.5 An 8-inch pipe contains a short section in which the diameter is gradually reduced to a 3-inch and then gradually enlarged to full size. If the pressure of water passing through is 75 lb. per sq. in. at a point just before the reduction commences, what will it be at the 3-inch section when the rate of flow is 1.2 cu. ft. per sec.? Assume no loss in head between the sections considered.

Ans. 71.0 lb. per sq. in.

2.6 A 12-inch pipe discharges water at a rate of 5.5 cu. ft. per second. At a section A of the pipe. the pressure is 40 psi while at section B. at a point where the pipe is 8 ft. lower than at A, the pressure is 42.5 psi. Compute the head loss between A and B.

Ans. 2.25 ft.

2.7 A horizontal pipe 12 in. in diameter carries water with a mean velocity of 10 ft. per second. At a section A, the pressure is 55 psi and at section B it is 40 lbs. per sq. in. Compute the quantity of energy passing each one of these sections in one second, estimating the potential energy with reference to a datum plane through the pipe's axis. What is the head lost between the two sections? Compute energy in ft. lbs. and hp.

Ans a. 62, 729 ft. lbs.

b. 45,829 ft. lbs.

c. 34.5 ft.

2.8 Water enters a motor through a 4-inch pipe under a gauge pressure of 150 psi. It leaves by an 8-inch pipe at an elevation 3 ft. below the point of entrance. If the pressure in the pipe at the exit is 10 psi and the discharge is 2 cu. ft. per sec., find the energy given up by the water each second as it passes the motor.

Ans 41,700 ft. lb.

CHAPTER III

Friction Losses in Pipes and Open Channels

3.1 Velocity and Flow Rates in Pipes Using the Hazen-Williams Formula

Flow in pipes can be either laminar or turbulent. In laminar flow the liquid might be said to be flowing in layers with each layer having its own velocity. Velocity is greatest at the center and least at the pipe surface. In laminar flow the main resistance to flow is friction between the layers. Laminar flow is very slow and its characteristics are quite orderly. Head loss can be determined by the equation:

$$HL = \frac{32 \nu L V}{r D^2}$$

It can be seen that the friction loss is dependent on the viscosity, length of pipe, average velocity, diameter of the pipe, and specific weight of the liquid. This equation gives good results for strickly laminar flow.

In turbulent flow the particles may be said to be "tumbling over one another" in a very irregular pattern. Motion is unpredictable and no simple calculation can be developed either from theoretical or practical methods. Almost all flow in pipes found in industry and commerce is turbulent.

Various individuals have developed equations experimentally for friction losses in pipes. These equations have friction factors that depend on the nature of the pipe and its age. Other factors are velocity and diameter of the pipe. All of the factors, having various exponential values, equate to the friction loss. Some of the equations that have been developed are Darcy, Chezy, and Hazen-Williams. The Hazen-Williams formula and accompanying friction factors will be used for this presentation.

FRICTION FACTORS

Type of Pipe	Coefficient C
New cast iron	130
Old cast iron	100
Finished wood and masonry	120
Vitrified clay	110

Hazen-Williams Formula For Pipe Losses

$$Q = 1.318\, C\, A\, R^{0.63} S^{0.54}$$

$$Q = 0.432\, C\, D^{2.63} S^{0.54}$$

Q = Flow in ft^3/sec

C = Friction Coefficient

A = Pipe Area ft^2

R = hydraulic radius = A/P

P = wetted perimeter

S = hydraulic slope = h_L/L

h_L = friction loss

L = Length of pipe

The term hydraulic radius is used to designate the term A/P. It is significant for flow in channels but for pipes flowing full it is usually more convenient to use the modified equation in terms of the diameter. Quite often the L term is given a value of 1. The h_L term will then be the friction loss per foot of pipe. Older pipes will have lower values due to changed conditions of the pipe surface. PVC pipe is often used today. It has good friction coefficients, some literature showing it to be as high as 140.

EX 3.1.1

An old cast iron pipe is 9" in diameter and 5000' long. Its flow is 5 fps. what is the friction loss?

$A = \frac{\pi}{4}(0.75)^2 = 0.442\ ft^2$

$Q = AV = (0.442)(5) = 2.21\ ft^3/sec$

$Q = 0.432\ C \cdot D^{2.63} \cdot S^{0.54}$

For 1' length $h_L/L = h_L/1 = S$

$2.21 = (0.432)(100)(0.75)^{2.63} \cdot h_L^{0.54}$

$2.21 = 43.2(0.469)(h_L)^{0.54}$

$h_L^{0.54} = 2.21 / 20.26 = 0.109$

$(h_L^{0.54})^{\frac{1}{0.54}} = (0.109)^{\frac{1}{0.54}} = (0.109)^{1.85}$

$h_L = 0.0166'$ per ft

Total $h_L = (0.0166)\ 5000 = 83'$ Ans.

EX 3.1.2

A 6" PVC pipe has a flow of 1 ft^3/sec. It has a length of 1000' and head loss has been found to be 15.5'. Determine the C value of the pipe.

$$Q = 0.432\, C\, D^{2.63} \cdot S^{0.54}$$

$$h_L/ft = 15.5/1000 = 0155'/ft$$

$$1 = 0.432\,(C)(05)^{2.63}(.0155)^{0.54}$$

$$1 = 0.432(C)(0.162)(0.1054)$$

$$1 = 007348\ C$$

$$C = 136.1 \qquad \text{Ans.}$$

3.2 Velocity and Flow Rates in Open Channels Using the Manning Equation

As in the case of flow in pipes it is not possible to develop simple equations for flow in open channels. Kutter, Bazin and Manning as well as others have developed equations experimentally. The Manning equation will be used in this presentation.

Open channels may be artificial or natural. Equations for natural channels are not as reliable as for artificial channels because of widely changing section and flow conditions. In artificial channels steady flow is assumed and when this condition is approached equations are quite accurate. Steady flow means that for all sections depth, mean velocity, and shape of channel are the same.

Unlike pipes, flow in channels depends entirely on the slope of the channel. The term S in this case stands for slope. $S = h/L$ where h is the vertical fall occurring in the length of the channel and L is the length of the channel.

$$v = \frac{1.486}{n} R^{\frac{2}{3}} \cdot S^{\frac{1}{2}} \quad \text{in ft/sec}$$

$$v = \frac{1.486}{n} \cdot R^{\frac{1}{6}} \sqrt{RS} = C\sqrt{RS}$$

$$C = \frac{1.486}{n} R^{\frac{1}{6}}$$

n = coefficient of roughness

$R = A/P$ = hydraulic radius

Coefficient C, By The Manning Formula

n	Hydraulic Radius in Feet											
	0.1	0.2	0.6	1.0	1.5	2	3	4	6	8	10	12
0.010	101	114	137	149	159	167	179	187	200	210	218	225
0.011	92	103	124	135	145	152	162	170	182	191	198	204
0.012	84	95	114	124	133	139	149	156	167	175	182	188
0.013	78	87	105	114	122	128	137	144	154	162	168	173
0.014	72	81	97	106	114	119	127	134	143	150	156	161
0.015	68	76	91	99	106	111	119	125	134	140	145	150
0.016	63	71	85	93	99	104	111	117	125	131	136	141
0.017	60	67	80	87	94	98	105	110	118	124	128	132

EX 3.2.1

A circular brick conduit 3 ft in diameter flows half full with a slope of 1 in 2000. Compute its probable discharge using the Manning Coefficient. ($n = 0.015$) $S = 1/2000 = 0.0005$

$A = \pi r^2/2 = \pi \cdot (1.5)^2/2 = 3.53 \text{ ft}^2$

$P = 2\pi r/2 = \pi r$

$R = \pi r^2/2 \div \pi r = r/2 = .75'$

$C = 1.486 R^{1/6}/n = 1.486 \cdot (.75)^{1/6}$

$= 94.4$

$V = C\sqrt{R \cdot S}$

$V = 94.4\sqrt{(0.75)(0.0005)}$

$V = 94.4(0.0194)$

$V = 1.828 \text{ ft/sec}$

$Q = AV = 3.53(1.828) = 6.45 \text{ cu ft/sec}$

Ans.

EX 3.2.2

. What slope would it be necessary to give a rectangular channel 50 ft. wide and 10 ft. deep in order to discharge 1500 cu ft. per sec.? Assume ordinary timber lining ($n = 0.012$)

$A = 50 \times 10 = 500 \text{ ft}^2$

$P = 50 + 10 + 10 = 70'$

$R = 500/70 = 7.14'$

$n = 0.012$ Use chart to find C

$C = 167 + 1.2(8)/2 = 171.8$

$V = Q/A = 1500/500 = 3 \text{ ft/sec}$

$V = C\sqrt{RS}$

$3 = 171.8\sqrt{7.14} \cdot \sqrt{S}$

$3 = 459.06 \cdot \sqrt{S}$

$\sqrt{S} = 3/459.06 = 0.0065$

$S = 0.000042$ Ans

3.3 Minor Losses

Obstructions to flow and changes in the pattern of flow in pipes cause friction losses. Examples of these losses are bends in pipes, valves, pipe expansion and contraction, and entrance and exit losses. These losses have been determined experimentally and are expressed as coefficients K of $\frac{v^2}{2g}$. A few commonly accepted values of K are listed below.

Ordinarily, for long lengths of pipes, these losses are small compared to pipe friction losses. However, for items such as globe valves, losses for several of them can be significant. The importantant thing in these considerations is how much the flow of water is affected. The wide difference in factors for gate and globe valves should be noted.

Item	Coefficient K
Entrance loss (rounded)	0.25
Exit loss	1.0
Check Valve	8.0
Globe Valve	10.0
90° elbow	0.3
45° elbow	0.2
Gate Valve	0.2

EX 3.3.1 Find losses for a globe valve with velocities of (a) 1, (b) 2, and (c) 5 fps respectively.

(a) $h_L = 10 \cdot V^2/2g = 10 \cdot (1)^2/64.4 = 10(.0155) = 0.155'$

(b) $h_L = 10 \cdot V^2/2g = 10(2)^2/64.4 = 10(0.063) = 0.63'$

(c) $h_L = 10 \cdot V^2/2g = 10(5)^2/64.4 = 10(0.388) = 3.88'$

EX 3.3.2 A one mile pipe has two 90° bends and one 45° bend. What is the total of these losses if the velocity is 5 fps ?

$V^2/2g = 5^2/64.4 = 0.388$

(a) 90° losses $= 2(.3)(0.388) = 0.233'$

(b) 45° losses $= 0.2(0.388) = 0.078'$

Total minor losses $= 0.311'$

EX 3.3.3 Determine losses for a gate valve with velocities shown in EX problem 3.3.3.

(a) $h_L = 0.2(0.0155) = 0.003'$

(b) $h_L = 0.2(0.063) = 0.012'$

(c) $h_L = 0.2(0.388) = 0.078'$

Chapter III Problems

3.1 Flow in a pipe is 5 fps. The entrance into the pipe is rough, having a K value of 0.80. What is the head loss? $h_L = 0.31'$ Ans

3.2 Water is flowing in a 1 diameter vitrified clay pipe. Velocity is 5 fps. What is the head loss for 1000 ft. of the pipe? $h_L = \text{total} = 10'$ Ans

3.3 Head loss for a 1.5' diameter pipe is 28.4 feet. The pipe is 3,940 ft. long and water flows at a rate of 8.8 ft^3/s. Find the friction constant. (C) Ans C=100

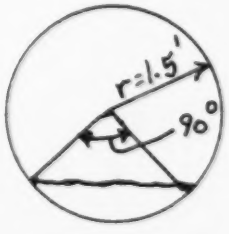
3.4 Flow is 3 fps. Determine the head loss for: (a) check valve, (b) 90° elbow, (c) globe valve, and (d) 45° elbow.

3.5 An old cast iron pipe is one mile long. Its velocity is 5 fps and its diameter is one foot. It has 4-90° elbows and 3-45° elbows. K for entrance is 0.82. The water passes one gate valve. Estimate the total head loss. $h_L = 64'$ Ans

3.6 Derive the Hazen-Williams pipe friction formula in terms of the diameter from the one in terms of the hydraulic radius.

3.7 Using the channel described in EX 3.2.2, what slope would be necessary for a discharge of 2500 ft per second? S = 0.00012 Ans

3.8 Flow in a 3' diameter brick conduit extends an arc of 90° as shown in the sketch. If n 0.015 and the slope is 1 in 1000 determine the probable discharge. $Q = 0.938\ ft^3/sec$ Ans



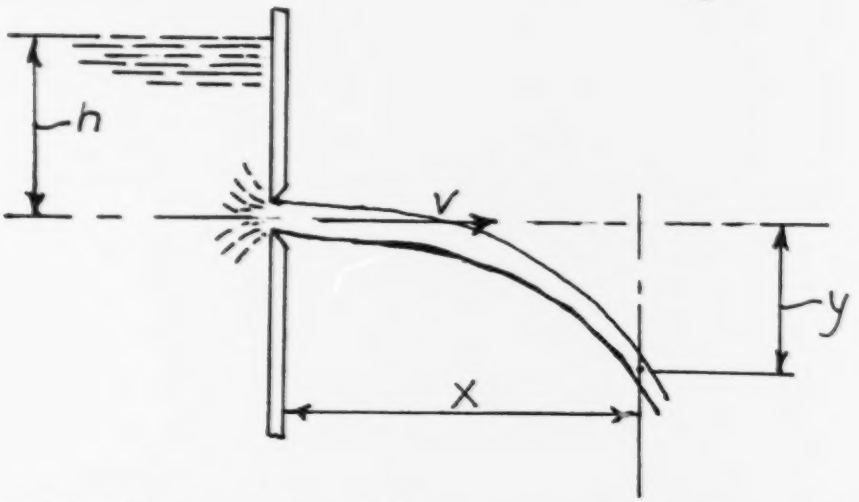
Chapter IV

Orifices

4.1 Basic Considerations

The term orifice refers to an opening in a wall or partition having a closed perimeter. Usually orifices studied have sharp edges. The effect of the orifice reduces the theoretical velocity of the water, the diameter of the stream compared to size of the opening, and the theoretical rate of discharge. Velocity and rate of discharge will be covered here.

4.2 Coefficients of Velocity



The theoretical velocity if there were no friction losses would be $\sqrt{2gh}$. The actual rate of velocity can be said to be $v = c_v\sqrt{2gh}$ where c_v is the coefficient of velocity. Neglecting the effect of air friction the horizontal distance covered will be $x = vt$. The y distance will be $gt^2/2$, using the equation for free falling bodies. Eliminating t, an equation for velocity in terms of x and y can be found. Using this value of v for a given orifice and head, the c_v value can be determined. With this basic equation, a formula for lost head can be developed.

$$V = C_v \sqrt{2gh}$$

$$\frac{V^2}{2g} = \text{head of stream}$$

$$\frac{V^2}{2g} = \frac{C_v^2\, 2gh}{2g} = C_v^2 h$$

$$\text{Lost head} = h - C_v^2 h = h(1 - C_v^2)$$

4.3 Coefficients of Discharge

The basic equation for flow is $Q = AV$ and this will be $A\sqrt{2gh}$ in terms of the theoretical velocity. Actual discharge will be $Q = A\sqrt{2gh}\, c_d$, c_d being the coefficient of discharge. Actual discharges of orifices can be easily and accurately determined. Numerous studies have been made to determine coefficients of discharge. A portion of these values for round orifices determined by Mendaugh and Johnson are shown below.

COEFFICIENTS OF DISCHARGE

Head in feet	Diameter of orifice in inches					
	0.25	0.50	0.75	1.00	2.00	4.00
0.8	0.647	0.627	0.616	0.609	0.603	0.601
1.4	0.635	0.619	0.610	0.605	0.601	0.599
2.0	0.629	0.615	0.607	0.603	0.600	0.599
4.0	0.621	0.609	0.603	0.600	0.598	0.597
6.0	0.617	0.607	0.601	0.599	0.596	0.596
8.0	0.614	0.605	0.600	0.598	0.596	0.595
10.0	0.613	0.604	0.599	0.597	0.595	0.595
12.0	0.612	0.603	0.599	0.597	0.595	0.595
14.0	0.611	0.603	0.598	0.596	0.595	0.594
16.0	0.610	0.602	0.598	0.596	0.595	0.594
20.0	0.609	0.602	0.598	0.596	0.595	0.594
25.0	0.608	0.601	0.597	0.595	0.594	0.594
30.0	0.607	0.600	0.597	0.595	0.594	0.594
40.0	0.606	0.600	0.596	0.595	0.594	0.593
50.0	0.605	0.599	0.596	0.595	0.594	0.593
60.0	0.605	0.599	0.596	0.594	0.593	0.593
80.0	0.604	0.598	0.595	0.594	0.593	0.593
100.0	0.604	0.598	0.595	0.594	0.593	0.593
120.0	0.603	0.598	0.595	0.594	0.593	0.592

EX 4.1 A 1" ϕ orifice has a coefficient of velocity of 0.95. Determine (a) the velocity and (b) the lost head under a head of 10 ft.

(a) $V = 0.95\sqrt{2gh}$

$= 0.95\sqrt{64.4(10)}$

$= 24.1$ fps

(b) $h_L = h[1 - Cv^2]$

$h_L = 10[1 - (0.95)^2]$

$h_L = 10[1 - 0.903] = 0.97'$

EX 4.2 A 1 ϕ orifice is under a head of 12 ft. Determine the flow quantity.

$A = \pi(0.0833)^2 = 0.0218 \ ft^2$

$Q = A\sqrt{2gh} : C_d$

$Q = 0.0218\sqrt{64.4 \cdot 12}\,(0.597)$

$Q = 0.0218\,(27.8)(0.597)$

$Q = 0.362 \ ft^3/sec$

Problems

4.1 An orifice has a c value of 0.97. The orifice size is $1''\phi$. Find (a) velocities and (b) lost heads under heads of 10 and 20 feet.

4.2 An orifice flows under heads of 10 and 20 feet. Find the discharge rates using the Mendaugh and Johnson coefficients if its size is $1''\phi$.